Q 13.1 (a) Lithium has two stable isotopes ^6_3Li and ^7_3Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, $^{10}_5B$ and $^{11}_5B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u.

Find the abundances of $^{10}_5B$ and $^{11}_5B$.

A1 :

(a) Mass of ^6_3Li lithium isotope , m1 = 6.01512 u

Mass of ^7_3Li lithium isotope , m = 7.01600 u

Abundance of ^6_3Li , η1= 7.5%

Abundance of ^7_3Li , η2= 92.5%

The atomic mass of lithium atom is:

$m = \frac{m_1n_1+m_2n_2}{n_1+n_2}$ $m = \frac{6.01512\times7.5+7.1600\times92.5}{92.5+7.5}$

= 6.940934 u

(b) Mass of boron isotope $^{10}_5B$ m = 10.01294 u

Mass of boron isotope $^{11}_5B$ m = 11.00931 u

Abundance of $^{10}_5B$, η1 = x%

Abundance of $^{11}_5B$, η2= (100 – x)%

Atomic mass of boron, m = 10.811 u
The atomic mass of boron atom is:

$m = \frac{m_1n_1+m_2n_2}{n_1+n_2}$ $10.811 = \frac{10.01294\times x+11.00931\times(100-x)}{x+100-x}$

1081.11 = 10.01294x + 1100.931 − 11.00931 x

x = 19.821/0.99637 = 19.89 %

And 100 – x = 80.11%

Hence, the abundance of $^{10}_5B$ is 19.89% and that of $^{11}_5B$ is 80.11%.

Q 13.2 :The three stable isotopes of neon: $^{20}_{10}Ne$, $^{21}_{10}Ne$ and $^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.
Answer

Atomic mass of $^{20}_{10}Ne$, m1= 19.99 u

Abundance of ${}^{20}_{10}Ne$, η1 = 90.51%

Atomic mass of ${}^{21}_{10}Ne$, m2 = 20.99 u

Abundance of ${}^{21}_{10}Ne$, η2 = 0.27%

Atomic mass of ${}^{22}_{10}Ne$, m3 = 21.99 u

Abundance of ${}^{22}_{10}Ne$, η3 = 9.22%

Below is the average atomic mass of neon:

$$m = \frac{m_1 n_1 + m_2 n_2 + m_3 n_3}{n_1 + n_2 + n_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

= 20.1771 u

Q 13.3: Obtain the binding energy in MeV of a nitrogen nucleus ${}^{14}_{7}N$, given m(${}^{14}_{7}N$)=14.00307 u

Ans:

Atomic mass of nitrogen ${}^{14}_{7}N$, m = 14.00307 u

A nucleus of ${}^{14}_{7}N$ nitrogen contains 7 neutrons and 7 protons.

$\Delta m = 7m_H + 7m_n - m$ is the mass defect the nucleus

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

Δm = 7 × 1.007825 + 7 × 1.008665 – 14.00307

= 7.054775 + 7.06055 – 14.00307

= 0.11236 u

But 1 u = 931.5 MeV/c^2

Δm = 0.11236 × 931.5 MeV/c^2

$E_b = \Delta mc^2$ is the binding energy of the nucleus

Where, c =Speed of light

$$E_b = 0.11236 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$$

= 104.66334 Mev

Therefore, 104.66334 MeV is the binding energy of the nitrogen nucleus.

Q 13.4: Obtain the binding energy of the nuclei ${}^{56}_{23}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data:

m (${}^{56}_{23}Fe$) = 55.934939 u

m(${}^{209}_{83}Bi$) = 208.980388 u

Ans :Atomic mass of ${}^{56}_{23}Fe$, m_1 = 55.934939 u

$^{56}_{23}Fe$ nucleus has 26 protons and (56 – 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m1$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

= 26.20345 + 30.25995 – 55.934939

= 0.528461 u

But 1 u = 931.5 MeV/c^2

$\Delta m = 0.528461 \times 931.5$ MeV/c^2

$Eb1 = \Delta mc^2$ is the binding energy of the nucleus.

Where, c = Speed of light

$$E_b = 0.528461 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$$

= 492.26 MeV

Average binding energy per nucleon = $\frac{492.26}{56} = 8.79 MeV$

Atomic mass of $^{209}_{83}Bi$, m_2 = 208.980388 u

$^{209}_{83}Bi$ nucleus has 83 protons and (209 – 83) 126 neutrons.

The mass defect of the nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

= 83.649475 + 127.091790 – 208.980388

= 1.760877 u

But 1 u = 931.5 MeV/c^2

$\Delta m'$ = 1.760877 × 931.5 MeV/c2

$Eb_2 = \Delta m'c^2$ is the binding energy of the nucleus.

= 1.760877 × 931.5 $\left(\frac{MeV}{c^2}\right) \times c^2$

= 1640.26 MeV

Average binding energy per nucleon = 1640.26/209 = 7.848 MeV

Q 13.5: A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ with mass = 62.92960 u.

Ans :

Mass of a copper coin, m' = 3 g

Atomic mass of $^{63}_{29}Cu$ atom, m = 62.92960 u

The total number of $^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{Mass\ number}$

Where,

NA = Avogadro's number = 6.023×10^{23} atoms /g

Mass number = 63 g

$N = \frac{6.023 \times 10^{23} \times 3}{63}$ = 2.868×10^{22} atoms

${}^{63}_{29}Cu$ nucleus has 29 protons and (63 – 29) 34 neutrons

Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m'$ = 29 × 1.007825 + 34 × 1.008665 – 62.9296

= 0.591935 u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

= $1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\Delta m = 1.69766958 \times 10^{22} \times 931.5\ MeV/c^2$

$E_b = \Delta mc^2$ is the binding energy of the nuclei of the coin

= $1.69766958 \times 10^{22} \times 931.5$

= 1.581×10^{25} MeV

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

= 2.5296×10^{12} J

Therefore, the energy required to separate all the neutrons and protons from the given coin is 2.5296×10^{12} J

Q 13.6: Write nuclear reaction equations for

(i) α-decay of ${}^{226}_{88}Ra$ *(ii) α-decay of* ${}^{242}_{94}Pu$

(iii) β−-decay of ${}^{32}_{15}P$ *(iv) β -decay of* ${}^{210}_{83}Bi$

(v) β+-decay of ${}^{11}_{6}C$ *(vi) β -decay of* ${}^{97}_{43}Tc$

(vii) Electron capture of ${}^{120}_{54}Xe$

Ans:

In helium, α is a nucleus ${}^{4}_{2}He$ and β is an electron (e− for β− and e+ for β+). 2 protons and 4 neutrons is lost in every α decay. Whereas 1 proton and a neutrino is emitted from the nucleus in every β + decay. In every β – decay, there is a gain of 1 proton and an anti-neutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$

(ii) ${}^{242}_{94}Pu \rightarrow {}^{238}_{92}U + {}^{4}_{2}He$

(iii) ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \bar{v}$

(iv) ${}^{210}_{83}B \rightarrow {}^{210}_{84}PO + e^- + \bar{v}$

(v) ${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + v$

(vi) ${}^{97}_{43}Tc \rightarrow \ {}^{97}_{42}MO \ + \ \ e^{+} + \ \ v$

(vii) ${}^{120}_{54}Xe + \ \ e^{+} \rightarrow \ \ {}^{120}_{53}I \ + \ \ v$

Q 13.7: A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Ans:

The half-life of the radioactive isotope = T years

N_0 is the actual amount of radioactive isotope.

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} \ = \ 3.125\% \ = \ \frac{3.125}{100} \ = \ \frac{1}{32}$$

But, $\frac{N}{N_0} \ = \ e^{-\lambda t}$

Where, λ = Decay

Constant t = Time

$$e^{-\lambda t} \ = \ \frac{1}{32} \ \ -\lambda t \ = \ ln1 – ln32 \ \ -\lambda t \ = \ 0–3.4657$$

Since, $-\lambda \ = \ \frac{0.693}{T}$

t = $\frac{3.466}{\frac{0.693}{T}} \ \approx \ \ 5T years$

Hence, the isotope will take about 5T years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} \ = \ 1\% = \ \frac{1}{100} \ \ \frac{N}{N_0} \ = \ e^{-\lambda t} \ \ e^{-\lambda t} \ = \ \frac{1}{100} \ \ -\lambda t = \ ln1 – ln100 \ \ e^{-\lambda t} = 0 - 4.6052$$

t = 4.6052/λ

Since, λ = 0.693/T

$t \ = \ \frac{4.6052}{\frac{0.693}{T}}$ *= 6.645 T years*

Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.

Q 13.8: The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ ***present with the stable carbon isotope*** ${}^{12}_{6}C$ ***. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of*** ${}^{14}_{6}C$ ***, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of*** ${}^{14}_{6}C$ ***dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.***

Ans :

Decay rate of living carbon-containing matter, R = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of ${}^{14}_{6}C, T_{\frac{1}{2}}$ *= 5730 years*

The decay rate of the specimen obtained from the Mohenjo-Daro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjo-Daro period.

Therefore, the relation between the decay constant, λ and time, t is:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t} \quad e^{-\lambda t} = \frac{9}{15} = \frac{3}{5} \quad -\lambda t = \log_e \frac{3}{5} = -0.5108 \quad t = \frac{0.5108}{\lambda}$$

But $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{5730} \quad t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5 years$

So, the approximate age of the Indus-Valley civilization is 4223.5 years.

Q 13.9: Obtain the amount of $^{60}_{27}Co$ ***necessary to provide a radioactive source***

of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ ***is 5.3 years.***

Ans:

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 mCi$$

= 8 x 10^{-3} *x 3.7 x* 10^{10}

= 29.6 x 10^{7} *decay/s*

Where,

N = Required number of atoms

Half-life of $^{60}_{27}Co$ *= 5.3 years*

= 5.3 × 365 × 24 × 60 × 60

= 1.67 × 10^{8} *s*

The rate of decay for decay constant λ is:

$$\frac{dN}{dt} = \lambda N$$

Where, $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{1.67 \times 10^{8}} s^{-1} \quad N = \frac{1}{\lambda}\frac{dN}{dt} = \frac{29.6 \times 10^{7}}{\frac{0.693}{1.67 \times 10^{8}}}$

= 7.133 x 10^{16} *atoms*

For $^{60}_{27}Co$ *:*

Mass of 6.023 × 10^{23} *(Avogadro's number) atoms = 60 g*

Mass of atoms 7.133 x 10^{16} *atoms = =* $\frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}}$

= 7.106 x 10^{-6} *g*

Hence, the amount of $^{60}_{27}Co$ *necessary for the purpose is 7.106 ×* 10^{-6} *g.*

Q 13.10 : The half-life of $^{90}_{38}Sr$ ***is 28 years. What is the disintegration rate of 15 mg of this isotope?***

Ans:

Half life of $^{90}_{38}Sr$ *,* $t_{\frac{1}{2}}$ *= 28 years*

= 28 × 365 × 24 × 60 × 60

= 8.83 × 10^{8} *s*

Mass of the isotope, m = 15 mg

90 g of $^{90}_{38}Sr$ *atom contains 6.023 ×* 10^{23} *(Avogadro's number) atoms.*

Therefore, 15 mg of $^{90}_{38}Sr$ contains :

$$\frac{6.023\times10^{23}\times15\times10\times10^{-3}}{90}$$

i.e 1.038 x 10^{20} number of atoms

Rate of disintegration, $\frac{dN}{dt} = \lambda N$

Where,

λ *= decay constant =* $\frac{0.693}{8.83\times10^{8}}s^{-1}$ $\frac{dN}{dt} = \frac{0.693\times1.0038\times10^{20}}{8.83\times10^{8}}$

= 7.878 x 10^{10} atoms/s

Hence, the disintegration rate of 15 mg of $^{90}_{38}Sr$ is

7.878 × 10^{10} atoms/s.

Q 13.11: Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the $^{107}_{47}Ag$ silver isotope .

Ans:

Nuclear radius of the gold isotope $^{197}_{79}Au$ = R_{Au}

Nuclear radius of the silver isotope $^{107}_{47}Ag$ = R_{Ag}

Mass number of gold, A_{Au} = 197
Mass number of silver, A_{Ag} = 107
Following is the relationship of the radii of the two nuclei and their mass number:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}} = \left(\frac{197}{107}\right)^{\frac{1}{3}}$$

= 1.2256
Hence, 1.23 is the ratio of the nuclear radii of the gold and silver isotopes.

Q 13.12: Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of

(a) $^{226}_{88}Ra$

(b) $^{220}_{86}Rn$

$m(^{226}_{88}Ra)$ ***= 226.02540 u,*** $m(^{222}_{86}Rn)$ ***= 222.01750 u,***

$m(^{220}_{86}Rn)$ ***= 220.01137 u,*** $m(^{216}_{84}Po)$ ***= 216.00189 u.***

Answer

(a) Alpha particle decay of $^{226}_{88}Ra$ emits a helium nucleus. As a result, its mass number reduces to 222 (226 – 4) and its atomic number reduces to 86 (88 – 2). This is shown in the following nuclear reaction.

$$^{226}_{88}Ra \rightarrow ^{222}_{86}Ra + ^{4}_{2}He$$

Q-value of emitted α-particle = (Sum of initial mass – Sum of final mass) c^2
Where, c = Speed of light
It is given that :

$m(^{226}_{88}Ra)$ *= 226.02540 u*

$m(^{222}_{86}Rn)$ = 222.01750 u

$m(^{4}_{2}He)$ = 4.002603 u

Q-value = [226.02540 – (222.01750 + 4.002603)] u c^2

= 0.005297 u c^2

But 1 u = 931.5 MeV/c^2

Q = 0.005297 × 931.5 ≈ 4.94 MeV

Kinetic energy of the α-particle = $\left(\frac{Mass\ number\ after\ decay}{Mass\ number\ before\ decay}\right) \times Q = \frac{222}{226} \times 4.94$ = 4.85 MeV

(b) Alpha particle decay of $^{220}_{86}Rn$

It is given that:

Mass of $^{220}_{86}Rn$ = 220.01137 u

Mass of $^{216}_{84}Po$ = 216.00189 u

Q-value =[220.01137 – (216.00189 + 4.00260)] x 931.5

≈ 641 MeV

Kinetic energy of the α-particle = $= \left(\frac{220-4}{220}\right) \times 6.41$

= 6.29 MeV

Q 13.13: The radionuclide ^{11}C decays according to $^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + v : T_{\frac{1}{2}} = 20.3\ min$. The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:

$m(^{11}_{6}C)$ **= 11.011434 u and** $m(^{11}_{5}B)$ **= 11.009305 u.**

Calculate Q and compare it with the maximum energy of the positron emitted

Ans:

The given nuclear reaction is:

$^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + v$

Half life of $^{11}_{6}C$ nuclei, $T_{1/2}$ = 20.3 min

Atomic mass of $m(^{11}_{6}C)$ = 11.011434 u

Atomic mass of $m(^{11}_{5}B)$ = 11.009305 u

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}C$ $\Delta Q = [m(^{11}_{6}C) - [m'(^{11}_{5}B + m_e)]]c^2$ --(1)

Where,

m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m′ = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 me in the case of ^{11}C and 5 me in the case of ^{11}B .

= 11.011434 u

Hence, equation (1) reduces to:

$$\Delta Q = [m(^{11}_{6}\mathrm{rm}C) – m(^{11}_{5}\mathrm{rm}B) - 2m_e]c^2$$

Here, $m(^{11}_{6}\mathrm{rm}C)$ and $m(^{11}_{5}\mathrm{rm}B)$ are the atomic masses.

ΔQ = [11.011434 – 11.009305 – 2 × 0.000548] c^2

= (0.001033 c^2) u

But 1 u = 931.5 Mev/c^2

ΔQ = 0.001033 × 931.5 ≈ 0.962 MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

Q 13.14: The nucleus $^{23}_{10}Ne$ decays β^- by emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m(^{23}_{10}Ne)$ **= 22.994466 u**

$m(^{23}_{11}Na)$ **= 22.989770 u.**

Ans:

In β^- emission, the number of protons increases by 1, and one electron and an antineutrino is emitted from the parent nucleus.

β^- emission from the nucleus $^{23}_{10}Ne$.

$$^{23}_{10}Ne \rightarrow ^{23}_{11} Na + e^- + \bar{\upsilon} + Q$$

It is given that:

Atomic mass of $m(^{23}_{10}Ne)$ = 22.994466 u

Atomic mass of $m(^{23}_{11}Na)$ = 22.989770 u

Mass of an electron, m_e = 0.000548 u

Q-value of the given reaction is given as:

$$Q = [m(^{23}_{10}Ne) – [m(^{23}_{11}Na) + m_e]]c^2$$

There are 10 electrons and 11 electrons in $^{23}_{10}Ne$ and $^{23}_{11}Na$ respectively. Hence, the mass of the electron is cancelled in the Q-value equation.

Q = [22.994466 – 22.989770] c^2

= (0.004696 c^2) u

But 1 u = 931.5 Mev/c^2

Q = 0.004696 x 931.5 = 4.374 MeV

The daughter nucleus is too heavy as compared to e^- and $\bar{\upsilon}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., 4.374 MeV.

Q 13.15: The Q value of a nuclear reaction A + b → C + d is defined by

Q = [m_A + m_b – m_C – m_d] c^2 where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions

are exothermic or endothermic.

(i) $^1_1H +^3_1\ H \rightarrow^2_1\ H +^2_1\ H$

(ii) $^{12}_6C +^{12}_6\ C \rightarrow^{20}_{10}\ Ne +^4_2\ H$

Atomic masses are given to be

$m(^2_1H) =\ \ 2.014102\ u$

$m(^3_1H) =\ \ 3.016049\ u$

$m(^{12}_6C) =\ \ 12.00000\ u$

$m(^{20}_{10}Ne) =\ \ 19.992439\ u$

Ans

(i) The given nuclear reaction is:

$^1_1H +^3_1\ H \rightarrow^2_1\ H +^2_1\ H$

It is given that:

Atomic mass $m(^1_1H) =\ \ 1.007825\ u$

Atomic mass $m(^3_1H) =\ \ 3.016049\ u$

Atomic mass $m(^2_1H) =\ \ 1.007825\ u$

According to the question, the Q-value of the reaction can be written as :

Q = [$m(^1_1H)$ *+* $m(^3_1H)$ *– 2m*$m(^2_1H)$ *]* c^2

= [1.007825 + 3.016049 – 2 x 2.014102] c^2

Q = (– 0.00433 c^2*) u*

But 1 u = 931.5 MeV/c^2

Q = -0.00433 x 931.5 = – 4.0334 MeV

The negative Q-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

$^{12}_6C +^{12}_6\ C \rightarrow^{20}_{10}\ Ne +^4_2\ H$

It is given that:

Atomic mass of $m(^{12}_6C) =\ \ 12.00000\ u$

Atomic mass of $m(^{20}_{10}Ne) =\ \ 19.992439\ u$

Atomic mass of $m(^4_2He) =\ \ 4.002603\ u$

The Q-value of this reaction is given as :

Q = [$m(^{12}_6C)$ *–* $m(^{20}_{10}Ne)$ *–* $m(^4_2He) =\ \ 4.002603\ u$ *]* c^2

= [2 x 12.0 – 19.992439 – 4.002603] c^2

= [0.004958 c^2] u

= 0.004958 x 931.5 = 4.618377 Mev

Since we obtained positive Q-value, it can be concluded that the reaction is exothermic.

Q 13.16: Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given

$$m(^{56}_{26}Fe) = \ 55.93494\ u$$

$$m(^{28}_{13}Al) = \ 27.98191\ u$$

Ans:

The fission of can $^{56}_{26}Fe$ be given as :

$^{56}_{26}Fe$ -> 2 $^{28}_{13}Al$

It is given that:

Atomic mass of $m(^{56}_{26}Fe) = \ 55.93494\ u$

Atomic mass of $m(^{28}_{13}Al) = \ 27.98191\ u$

The Q-value of this nuclear reaction is given as :

Q = [$m(^{56}_{26}\mathrm{rm}Fe) – 2m(^{28}_{13}\mathrm{rm}Al)$] c^2

= [55.93494 – 2 x 27.98191]c^2

= (-0.02888 c^2) u

But 1 u = 931.5 MeV/c^2

Q = – 0.02888 x 931.5 = -26.902 MeV

Since the Q-value is negative for the fission, it is energetically not possible.

Q 13.17: The fission properties $^{239}_{94}Pu$ of are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission ?

Ans:

Average energy released per fission of $^{239}_{94}Pu$, E_{av} = 180Mev

Amount of pure $^{239}_{94}Pu$, m = 1 kg = 1000 g

N_A= Avogadro number = 6.023 × 10^{23}

Mass number of $^{239}_{94}Pu$ = 239 g

1 mole of $^{239}_{94}Pu$ contains NA atoms.

Therefore, mg of $^{239}_{94}Pu$ contains $\left(\frac{N_A}{Mass\ Number} \times m\right)$ atoms

$\frac{6.023 \times 10^{23}}{239} \times \ 1000 = \ 2.52 \times \ 10^{24}$ atoms

Total energy released during the fission of 1 kg of ${}^{239}_{94}Pu$ is calculated as:

$E = E_{av} \times 2.52 \times 10^{24}$

$= 180 \times 2.52 \times 10^{24}$

$= 4.536 \times 10^{26}$ MeV

Hence, 4.536×10^{26} MeV is released if all the atoms in 1 kg of pure ${}^{239}_{94}Pu$ undergo fission.

Q 13.18: A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Answer

Half life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5$ years

$= 5 \times 365 \times 24 \times 60 \times 60$ s

We know that in the fission of 1 g of ${}^{235}_{92}U$ nucleus, the energy released is equal to 200MeV.

1 mole, i.e., 235 g of ${}^{235}_{92}U$ contains 6.023×10^{23} atoms.

1 g contains ${}^{235}_{92}U$ => $\frac{6.023\times10^{23}}{235}$ atoms contains

The total energy generated per gram of ${}^{235}_{92}U$ is calculated as:

$$E = \frac{6.023\times10^{23}}{235} \times 200MeV/g \text{ MeV/g}$$

$$= \frac{200\times6.023\times10^{23}\times1.6\times10^{-19}\times10^{6}}{235}$$

$= 8.20 \times 10^{16}$ J/g

The reactor operates only 80% of the time.

Hence, the amount of ${}^{235}_{92}U$ consumed in 5 years by the 1000 MW fission reactor is

calculated as:

$$= \frac{5\times80\times60\times60\times365\times24\times1000\times10^{6}}{100\times8.20\times10^{10}} \approx 1538 \text{ Kg}$$

Initial amount of ${}^{235}_{92}U$ = 2 × 1538 = 3076 kg

Q 13.19: How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium?

Take the fusion reaction as

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}He + n + 3.27MeV$$

Ans :

The given fusion reaction is:

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}He + n + 3.27MeV$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

2.0 kg of deuterium contains $= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{23}$

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} MeV \; E = \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

= 1.576 x 10^{14} *J*

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

= $\frac{1.576 \times 10^{14}}{100} s$

= $\frac{1.576 \times 10^{14}}{100} s$ *years*

Q 13.20: Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Ans :

When two deuterons collide head-on, the distance between their centers, d is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2 × 10^{-15} *m*

d = 2 × 10^{-15} *+ 2 ×* 10^{-15} *= 4 ×* 10^{-15} *m*

Charge on a deuteron nucleus = Charge on an electron = e = 1.6 × 10^{-19} *C*

Potential energy of the two-deuteron system:

$$v = \frac{e^2}{4\pi\epsilon_0 d}$$

Where,

ϵ_0 *= Permittivity of free space*

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^{9} Nm^2C^{-2}$$

Therefore,

$$V = \frac{9 \times 10^{9} \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} J \; V = \frac{9 \times 10^{9} \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15} \times (1.6 \times 10^{-19})} eV$$

= 360 keV

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

Q 13.21: From the relation R = $R_0A^{1/3}$***, where*** R_0 ***is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).***

Ans :

We have the expression for nuclear radius as:

R = $R_0A^{1/3}$

Where,

R_0 *= Constant.*

A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{Mass\ of\ the\ nucleus}{Volume\ of\ the\ nucleus}$

Consider m be the average mass of the nucleus.

Hence, mass of the nucleus = mA

Therefore , $\rho = \frac{mA}{\frac{4}{3}\pi R^3}$

= $\frac{3mA}{4\pi(R_0 A^{\frac{1}{3}})^3}$

= $\frac{3mA}{4\pi R_0^3 A}$

= $\frac{3m}{4\pi R_0^3}$

Since the nuclear matter density is independent of A, it is nearly constant.

Q 13.22: For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^A Y + v$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

Ans :

Let the amount of energy released during the electron capture process be Q1. The nuclear reaction can be written as:

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^A Y + v + Q_1 \quad \text{--(1)}$$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as:

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^A Y + v + Q_2 \quad \text{--(2)}$$

$$m_n({}_Z^A X) = \text{ Nuclear mass of } {}_Z^A X \;\; m_n({}_{Z-1}^A X) = \text{ Nuclear mass of } {}_{Z-1}^A Y \;\; m({}_Z^A X) = \text{ Atomic mass of } {}_Z^A X \;\; m({}_{Z-1}^A X) =$$

$$\text{Atomic mass of } {}_{Z-1}^A Y$$

m_e = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$Q_1 = [m_n({}_Z^A X) + m_e - m_n({}_{Z-1}^A Y)]c^2$

$= [m_n({}_Z^A X) - Zm_e + m_n({}_{Z-1}^A Y) - (Z-1)m_e]c^2$

$= [m_n({}_Z^A X) - m_n({}_{Z-1}^A Y)]c^2 \quad \text{--(3)}$

Q – value of the positron capture reaction is given as :

$Q_2 = [m_n({}_Z^A X) - m_n({}_{Z-1}^A Y) - m_e]c^2$

$= [m_n({}_Z^A X) - Zm_e - m_n({}_{Z-1}^A Y) + (Z-1)m_e - m_e]c^2$

$= [m_n({}_Z^A X) - m_n({}_{Z-1}^A Y) - 2m_e]c^2 \quad \text{--(4)}$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically allowed nuclear reaction.